

**Note: Assume any data required, state your assumption clearly.**  
**Answer only four questions**

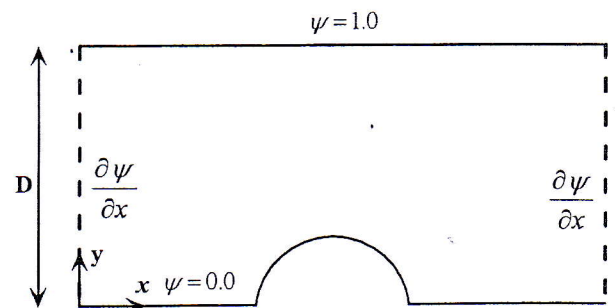
**Question (1) (25 Marks)**

The original x-momentum can be written with  $\eta$  as the independent variable and  $f$  is the dependent variable as  $2 \frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} = 0$  upon simplifications. Calculate the value of  $f$  over the range  $\eta = 0$  to 5 using a step size of 1 with  $f(0) = f'(0) = 0$  and  $f''(0) = 0.3321$

**Question (2) (25 Marks)**

The ideal incompressible flow around circular cylinder can be given by  $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ , where  $\psi$  is the stream function. Due to flow symmetry, only the upper half of the domain needs to be solved, see the figure. The cylinder center located at  $x = L/2$ . The velocity distribution can be calculated from  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ , where  $u$  and  $v$  are the horizontal and vertical velocity component, respectively. Using the boundary conditions shown in the figure, answer the following:

- Describe the solution procedure of this equation using finite difference method
- Write a computer program for the solution procedure described in a.
- Show how the velocity distribution can be obtained numerically

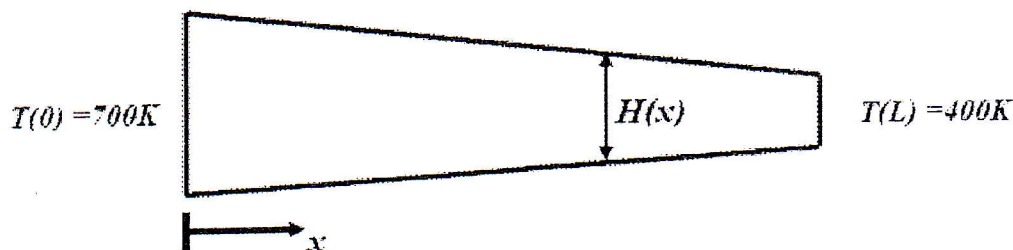


**Question (3) (25 Marks)**

The heat transfer equation in trapezoidal fin shown in the next figure is given by

$$\frac{\partial}{\partial x} \left( kA(x) \frac{\partial T}{\partial x} \right) + hP(x)(T - T_\infty) = 0$$

Where,  $k$  is the thermal conductivity,  $P(x)$  and  $A(x)$  are the perimeter and cross sectional area of the fin at any  $x$ . given that:  $k = 19 \text{ W/m.K}$ ,  $T_\infty = 300\text{K}$ ,  $h = 2 \text{ W/m}^2\text{K}$ , the fin length is 50 cm and fin width (perpendicular to paper) is 15 cm, the fin height is  $H(x) = 5 - 0.005x$  cm. Calculate the temperature distribution along the fin using five grid points



**Question (4) (25 Marks)**

Consider a fluid bounded by two parallel plates extended to infinity such that no end effects are encountered. The planar walls and the fluid are initially at rest. The lower wall is suddenly

accelerated in the x-direction with a velocity of 40 m/s. The kinematic viscosity of the fluid is  $0.000217 \text{ m}^2/\text{s}$  and the spacing between plates is 40 mm. The Navier-Stokes equation for this problem may be expressed as;  $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$  where  $\nu$  is the kinematic viscosity.

Describe the numerical solution to compute the velocity profiles  $u=u(t,y)$  by The FTCS explicit method with  $\Delta y=\Delta t=0.01$  march your solution for the FTCS explicit methods with only five spatial points and five time steps.

**Question (5)**

**(25 Marks)**

For the U-duct bend shown in the figure is composed from upstean and downstream ducts of length 10 and width 1 and a curved 180° bend of internal raduis of 2. Answer the following

- Use an elliptic grid generator to obtain body fitted domain.
- Describe in details the boundary conditions used.
- Obtain the transformation metrics
- Write computer program to obtain the transformation metrics

The following equations can be used

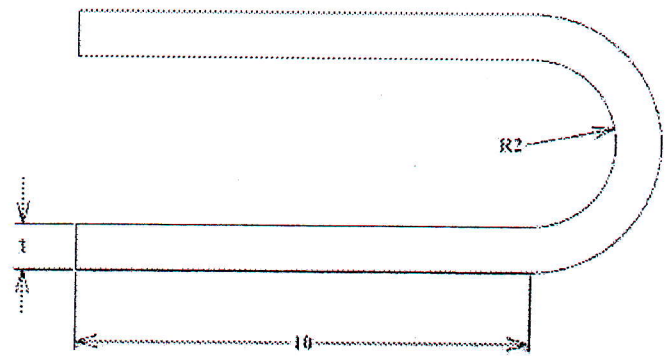
$$ax_{\zeta\zeta} - 2bx_{\zeta\eta} + cx_{\eta\eta} = 0$$

$$ay_{\zeta\zeta} - 2by_{\zeta\eta} + cy_{\eta\eta} = 0$$

$$a = x_{\eta}^2 + y_{\eta}^2$$

$$b = x_{\zeta}x_{\eta} + y_{\zeta}y_{\eta}$$

$$c = x_{\zeta}^2 + y_{\zeta}^2$$



**Question (6)**

**(25 Marks)**

The  $x$ - component of Navier-Stokes equation in two-dimensional with no body force can be written

as: 
$$\frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho uv}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

Transfer the above equation to body fitted coordinates

**GOOD LUCK**

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